GENERAL METHOD FOR TRANSFORMATION OF HEAT- AND MASS-TRANSFER PROCESSES IN A BED

G. A. Aksel'rud and Ya. T. Rogotskii

UDC 66.015.23.24.001.57

A general method is worked out for transforming the output curve (of the temperature or concentration) for a bed of width h_1 into the output curves for beds of any other width h_m, where $h_m \gtrless h_1$.

We are concerned here with certain questions which arise in connection with the heat and mass transfer as a liquid or gas moves through a bed of solid particles.

In the case of heat transfer, the bed is heated or cooled, and the gas or liquid moving through the bed is also heated or cooled; in the case of mass transfer, material is either extracted from or absorbed by the porous particles of the bed.

We use the term "output curve" to represent the time dependence of the temperature (in the case of heat transfer) or concentration (mass transfer) of the medium leaving the bed. There has been much theoretical work on these processes [1, 3, 4], but it has been based on several assumptions which are frequently dubious. For example, the particles have been assumed spherical, and the one-dimensional problem has been treated [3, 4]. This approach implies that the temperature or concentration within a particle is a function of the radius only. Such assumptions would be justified if the conditions under which the heat or material is extracted at the boundary of the sphere were the same at all points of the sphere. It is easy to see that this is not the case where the particles touch each other. Other important factors in heat and mass transfer are the inhomogeneity of the particles, the anisotropy, differences in particle size, and other factors, which have not been included in the existing theoretical models for these processes.

Since even the simplest models are complicated, it is an extremely difficult problem to match the model with the particular experiment and thus find the kinetic coefficients. These difficulties led to the suggestion that the information contained in the experimental output curve for a bed of width h_1 be used to predict processes in a bed of any other width h_m, where $h_m \gtrless h_1$ [1].

The general transformation method can be outlined as follows: if $f_1(t)$ and $f_m(t)$ are the output curves for beds of the same "quality" but different widths, then they are related by [1]

$$f_m(t) = L^{-1}\left\{\frac{1}{p}[pF_1(p)]^m\right\}, \tag{1}$$

where $F_1(p) = L[f_1(t)]$ is the Laplace transform of the function $f_1(t)$.

Let us assume that the output curve $f_1(t)$ has been found experimentally. For transformations in accordance with Eq. (1) the function $f_1(t)$ is approximated by a sum of exponential functions,

$$f_1(t) = 1 - \sum_{i=1}^{N} A_i \exp(-\mu_i t); \tag{2}$$

then a Laplace transformation yields

$$F_m(p) = \frac{1}{p}\left(1 - \sum_{i=1}^{N} \frac{A_i p}{p + \mu_i}\right)^m. \tag{3}$$

L'vov Polytechnical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 5, pp. 894-896, May, 1974. Original article submitted July 9, 1973.

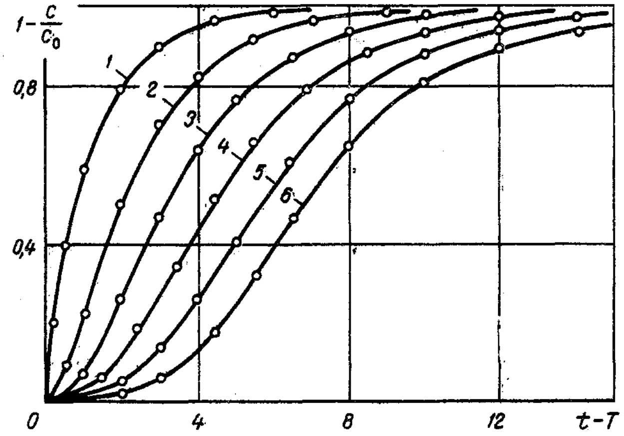

Fig. 1. Output concentration curves for beds of various widths: 1) $h_1 = 0.1$ m; 2) $h_2 = 0.2$; 3) $h_3 = 0.3$; 4) $h_4 = 0.4$; 5) $h_5 = 0.5$; 6) $h_6 = 0.6$. The curves show the results of the computer calculations, while the points are experimental.

An important part of this problem is to find the exact inverse transform of (3). Analytic methods (the expansion theorems, methods of contour integration, and tables of transforms) are not always adequate for this purpose. It therefore becomes necessary to use approximate methods for numerically finding the inverse transform.

We selected the Papoulis numerical method [2] for finding the inverse transform of (3) because of its simplicity and accuracy.

According to this method the inverse transform is expanded in a series of Legendre polynomials of argument $\exp(-\sigma t)$:

$$f_m(t) = \sum_{k=0}^{M} B_k P_{2k} \exp(-\sigma t), \tag{4}$$

where B_k are the expansion coefficients, determined from the transform $f_m(p)$ at the points $p = (2k + 1)\sigma$ on the basis of the system of equations given in [2].

We first studied the accuracy of this method and the choice of the parameters σ and M providing sufficient accuracy in finding the inverse transform.

Calculations were carried out on an M-222 computer. The inverse transforms found by this method agree within 0.01% with the exact transforms for aperiodic functions with $\sigma = 0.2$ and $M = 7$.

In related experiments we immersed particles of a porous clay filler in a 1% solution of sulfuric acid and held them there for 3 days until the pores were completely saturated with solution. Then beds of the porous particles with widths from $h_1 = 0.1$ m to $h_6 = 0.6$ m were formed in an extraction column.

Distilled water was filtered through a bed at a velocity of $v = 7 \cdot 10^{-4}$ m/sec, held constant throughout all the experiments. Samples of solution were taken from the rear of a bed at time intervals of $t = 30$ sec, and the solution concentration in each sample was determined with an TFL-46-2 titrator, with the neutralization point determined with a PH-340 pH meter.

Figure 1 (curve 1) shows the experimental output curve for a bed of width $h_1 = 0.1$ m, approximated by the sum of two exponential functions:

$$f_1(t) = 1 - 0.8\exp(-0.69t) - 0.2\exp(-2.76t). \tag{5}$$

Then Eq. (1) was used for computer calculations of the output curves for beds of widths $h_m = 2h_1$, $3h_1$, $4h_1$, $5h_1$, $6h_1$; these curves are also shown in Fig. 1 (curves 2-6, respectively).

The calculated data are compared with the experimental data in this figure.

Accordingly, this method seems useful for predicting the kinetics of the extraction of a dissolved substance from a bed of any width.

NOTATION

p	is the parameter of the Laplace transformation;
$m = h_m/h_1$	if the change in the bed width;
A_i, μ_i	are the coefficients;
P_{2k}	are the even Legendre polynomials;
c	is the solution concentration at the rear of the bed, %;
c_0	is the initial solution concentration, %;
t	is the time, min;
T	is the filtration time, min.

LITERATURE CITED

1. G. A. Aksel'rud, Mass Transfer in a Solid—Liquid System [in Russian], Izd. L'vovskogo Universiteta, L'vov (1970).
2. A. Papoulis, Quart. Appl. Math., 14, 405-414 (1957).
3. J. B. Rosen, J. Chem. Phys., 20, 387 (1952).
4. J. B. Rosen, Ind. Engn. Chem., 46, 1590 (1954).